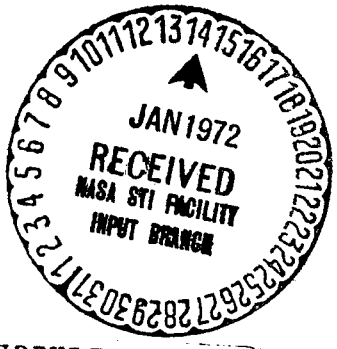
# EXPANSION OF A TURBULENT-JET BOUNDARY LAYER IN A SLIPSTREAM

V. K. Temnov

Translation of "O rasshirenii pogranichnogo sloya turbulentnoy strui vsputnom potoke," Izvestiya Vysshikh Uchebnykh Zavedeniy, Aviatsionnaya Tekhnika, Vol. 13, No. 4, 1970, pp. 111-115.




N72-13273 (ACCESSION NUMBER) (THRU)
(NASA-TT-F-14060) EXPANSION OF A TURBULENT JET BOUNDARY LAYER IN A SLIPSTREAM V.K. Temnov (Translation Consultants, Ltd.)
Unclas Dec. 1971 9 p CSCL 20D
11468 G3/12

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 DECEMBER 1971

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
Springfield, Va. 22151

# EXPANSION OF A TURBULENT-JET BOUNDARY LAYER IN A SLIPSTREAM

V. K. Temnov

ABSTRACT: A formula derived by Abramovich (1960) for the expansion of a boundary layer is checked experimentally, using circular and plane jets. A new formula for calculating the boundaries of the flow core is proposed which, in combination with the momentum equation, can be used to calculate the bounds of a jet boundary layer at the initial portion of the jet. An analytical expression describing the behavior of the axial velocity of the jet beyond its initial portion is obtained. A71-27505

## Designations

/111*

$b$ - width of jet boundary layer;

$r_0$, $b_0$ - radius and half-width of jet at the initial cross-section;

$y_1$ - radius and half-width of jet core;

$y$, $x$ - transverse and longitudinal coordinates;

$x_i$ - length of initial portion;

$\eta = \frac{y - y_1}{b}$ - relative transverse coordinate;

$u$, $v$ - longitudinal and transverse components of the velocity;

$v'$ - pulsation component;

$V$ - slipstream velocity;

$W$, $u_m$ - axial velocity of jet at the initial portion and beyond its limits;

$\varphi = \frac{u - V}{W - V}$ - relative excess velocity along the x axis;

$\mu = \frac{u_m - V}{W - V}$ - relative excess velocity along the jet axis;

$m = V/W$ - relative velocity of the slipstream;

$C$, $\alpha_1$, $\alpha_2$ - experimental coefficients.

1. G. N. Abramovich's formula [1]

$$\frac{db}{dx} = C\frac{1-m}{1+m}, \tag{1}$$

where

$b$ is the width of the jet boundary layer;

---

* Numbers in the margin indicate pagination in the foreign text.

x is the longitudinal coordinate;

m = V/W (V is slipstream velocity, W is axial velocity at the initial portion of the jet);

C is an experimental coefficient,

is widely used to calculate a turbulent jet propagating in the slipstream of an incompressible liquid.

This relationship is approximate in nature and subject to verification. It is impossible to measure the width of the boundary layer in experiments, so roundabout methods must be resorted to in order to check this formula. G. N. Abramovich has described one such method in his monograph [1]. This article will discuss the use of another method; along the length of the initial portion of the jet.

We shall limit our analysis to turbulent jets with a uniform initial field of velocities propagating in a slipstream without a pressure gradient. Integration of Eq. (1) reduces to the following formula [1] for the length of the initial portion:

(a) for a circular jet

/112

$$C\frac{x_i}{r_0} = \frac{1+m}{(1-m)\sqrt{0.214+0.144m}}; \quad (2)$$

(b) for a plane jet

$$C\frac{x_i}{b_0} = \frac{1+m}{(1-m)(0.416+0.133m)}. \quad (3)$$

where

$x_i$ is the length of the initial portion;

$r_0$, $b_0$ are radius and half-width of the jet at the initial cross-section.

These formulas can be used to find C if the length of the initial portion, and the relative velocity of the slipstream are known. The smooth conjunction of the initial and terminal portions makes it extremely difficult to measure $x_i$ directly, so the length of the initial portion was determined indirectly, from the axial velocity profile. The wealth of experimental data [1] - [6] available helped us select the approximating function for the profile of the jet's axial velocity. Figure 1 shows the longitudinal gradient of the inverse magnitude of the relative excess velocity along the axis of the circular jet in

terms of the relative longitudinal coordinate

$$\frac{d}{dx_*}\left(\frac{1}{\mu}\right) = f(x_*),$$

where

$x_* = x/r_0$, $\mu = \frac{u_m - V}{W - V}$ ($u_m$ is the axial velocity).

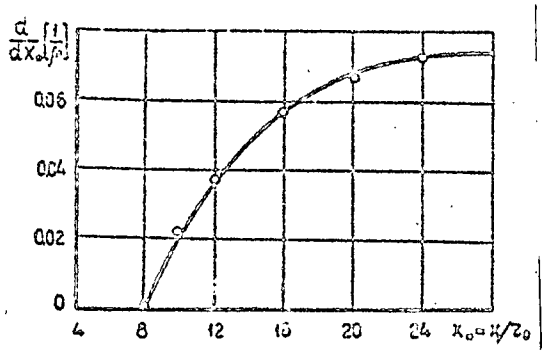



Figure 1.

The points show the values of this gradient, found experimentally [4], and the solid line is the approximating function of the hyperbolic tangent. Coincidence is quite satisfactory, so for the circular jet, it can be taken that

$$\frac{d}{dx_*}\left(\frac{1}{\mu}\right) = \alpha_2\sqrt{1-m}\,\mathrm{th}\left(\frac{x}{x_i} - 1\right). \qquad (4)$$

where

$\alpha_2$ is the "damping" factor for the axial velocity.

Integrating Eq. (4), and recognizing that at the boundary of the initial portion ( $x = x_i$) $\mu = 1$, we obtain

$$\frac{1}{\mu} = 1 + \alpha_2\sqrt{1-m}\,\frac{x_i}{r_0}\ln\mathrm{ch}\left(\frac{x}{x_i} - 1\right). \qquad (5)$$

For a plane jet

$$\frac{1}{\mu^2} = 1 + \alpha_2(1-m)\frac{x_i}{b_0}\ln\mathrm{ch}\left(\frac{x}{x_i} - 1\right). \qquad (6)$$

Superposing the Eq. (5) and Eq. (6) profiles on the experimental ones, we can find $\alpha_2$ and $x_i$. Figure 2 juxtaposes the calculated axial velocity profile of Eq. (5) and the experimental one found by I. B. Palatnik [3] for different relative slipstream velocities, and Figure 3 shows the values of C in G. N. Abramovich's formula, Eq. (1), calculated in terms of Eqs. (3) and (2). The results of the experiments conducted by I. B. Palatnik [3], E. Timma, Yu. Ivanov [2], F. Landis, A. Shapiro [1], I. F. Teylor, H. L. Grimmet, E. W. Comings [4], E. Förthmann [5], and A. S. Weinstein, I. F. Osterle, W. Forstall [6], were used to find C. /113

Figure 3 shows C in terms of relative slipstream velocity, and contradicts the very substance of Eq. (1), in which the influence of the relative slipstream

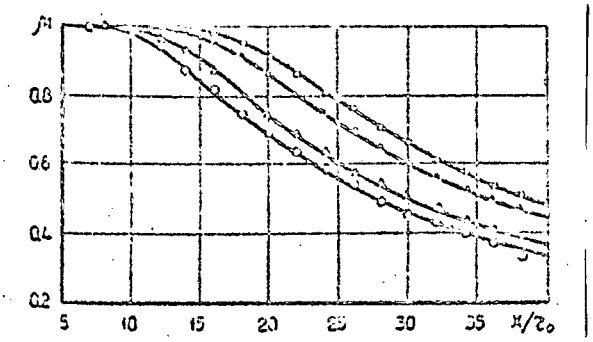



Figure 2.

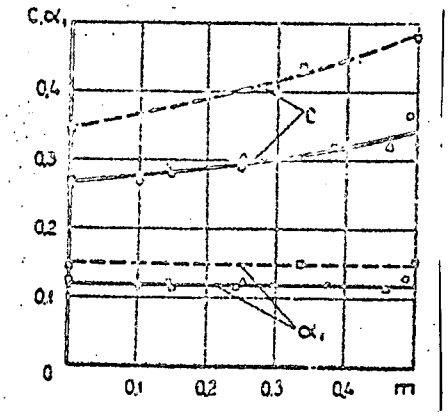



Figure 3.

velocity can be taken into consideration by the factor $(1 - m)/(1 + m)$, and the influence of all the other factors by C.

Thus, the experiments do not confirm the structure of Eq. (1).

2. As distinguished from G. N. Abramovich, let us consider separately the expansion of the jet boundary layer in the internal and external regions. The intensity of the expansion of the boundary layer in the internal region (inside the core of the jet) obviously is directly proportional to the pulsating component of the transverse velocity, v', and inversely proportional to the longitudinal velocity at the boundary layer - to the core velocity

$$\frac{dy_1}{dx} \sim \frac{v'}{W},$$

and

$$v' \sim l \frac{\partial u}{\partial y}.$$

where

y is the transverse coordinate reckoned from the jet axis;

$y_1$ is the radius, or the half-width of the jet core;

u is the longitudinal component of the velocity.

Assuming that the path of the shift, l, is proportional to the width of the boundary layer, and taking the maximum longitudinal velocity as the characteristic transverse gradient, we obtain /114

After progressing to equality, and to a more universal variable

[illegible] and [illegible]

we have

[illegible] (7)

If the velocity profiles at the boundary layer are affinely similar, $\varphi'_{max}$ = - constant. Consequently,

[illegible] (8)

m = constant in the absence of a longitudinal pressure gradient, so

[illegible] (9)

where

[illegible] (10)

These equations, can only be approximate for a circular jet, because the profiles $\varphi(\eta)$ are deformed along the initial portion, causing a change in $\varphi'_{max}$ of approximately 5.5 percent. The actual boundary of the core of a circular jet therefore is curved, whereas it is a straight line for the plane jet.

The correctness of the structure of Eq. (8) can be verified by using the same method. Figure 3 shows the results of the calculation of $\alpha_1$ by using Eq. (10), and including the same experiments [1] - [6]. As distinguished from C, $\alpha_1$ does not depend on the relative slipstream velocity, for all practical purposes, and this confirms the correctness of the structure of Eq. (8). The following values are obtained for the factors: for the circular jet $\alpha_1 = 0.12$, $\alpha_2 = 0.078$; for the plane jet $\alpha_1 = 0.15$, $\alpha_2 = 0.090$.

Thus, experiments with circular and plane jets are better confirmation of Eq. (8) than is Eq. (1). At the same time, Eq. (8) is very much simpler, and this is very important in solving different engineering problems through the use of jet theory.

The law of conservation of momentum can be used to calculate the expansion of the jet boundary layer in the external region. If the jet is propagated in

a homogeneous slipstream with infinite transverse dimensions, the momentum equation, in the projection on the jet axis, can be written [1]

$$W(W-V)(b_0-y_1)=\int_{y_1}^{y_2}u(u-V)\,dy \quad \text{(for a plane jet)}$$

$$W(W-V)(r_0^2-y_1^2)=2\int_{y_1}^{y_2}u(u-V)\,y\,dy \quad \text{(for a circular jet)}$$

where

$y_2$ is the transverse coordinate of the external boundary of the boundary layer.

Using the universal variables $\varphi$ and $\eta$, the equations can be given the form /115

$$(b_0-y_1)=(y_2-y_1)[k_1+(k_2-k_1)m], \qquad (11)$$

$$(r_0^2-y_1^2)=2(y_2-y_1)^2[k_1+(k_2-k_1)m]+2y_1(y_2-y_1)[k_2+$$

where

$$k_1=\int_0^1\varphi^2\eta\,d\eta,\quad k_2=\int_0^1\varphi^2\,d\eta,\quad k_3=\int_0^1\varphi\cdot\,d\eta,\quad k_4=\int_0^1\varphi\,d\eta.$$

Reproduced from best available copy.

The numerical values of the integral characteristics, $k_i$, are determined by the profiles of the longitudinal velocities in the boundary layer. When the profiles have $\varphi = 1 - 10\eta^3 + 15\eta^4 - 6\eta^5$ (plane jet) and $\varphi = 1 - 20\eta^3 + 45\eta^4 - 36\eta^5 + 10\eta^6$ (circular jet), which agree with the experiment within the limits of the jet's initial portion better than the other known power profiles, the integral characteristics are equal to

$k_2=0.392$; $k_4=0.500$ (plane jet);
$k_1=0.085$; $k_2=0.329$; $k_3=0.107$;
$k_4=0.425$ (circular jet).

Eq. (11), with an assist from Eq. (9), provides the formulas for calculating the external bound of the boundary layer at the initial portion

(a) for a plane jet

$$\frac{y_2}{b_0}=1+\frac{0.608-0.[illegible]m}{0.[illegible]2+0.[illegible]m}\frac{x}{x_i};$$

(b) for a circular jet

$$\frac{y_2}{r_0}=10\sqrt{\frac{10}{126+88m}-\frac{175+22[illegible]-10[illegible]m^2}{(126+88m)^2}\left[1-\frac{x}{x_i}\right]^2}-\frac{203+12m}{126+88m}\left[1-\frac{x}{x_i}\right].$$

where

$x$ is the actual distance from the nozzle;

$x_i$ is the length of the initial portion, found through Eq. (10).

REFERENCES


1. Abramovich, G. N., Teoriya turbulentnykh struy [Theory of Turbulent Jets], Fizmatgiz, 1960.

2. Timma, E., Ivanov, Yu., "Circular and Plane Turbulent Jets Developing in a Slipstream," Izv. AN Est. SSSR., T. XI. ser. fiz-mat. i. tekhnicheskikh nauk, No. 3, 1962.

3. Palatnik, I. B., "On the Expansion of an Axisymmetrical Turbulent Jet of Finite Size in a Homogeneous Slipstream," Izv. AN Kaz. SSSR, ser. energeticheskaya, vol. 2 (18), 1960.

4. Teylor, I. F., Grimmett, H. L., Comings, E. W., "Isothermal Free Jets of Air Mixing with Air," Chemical Eng. Progress, Vol. XLVII, No. 4, 1951.

5. Förthmann, E., "Turbulent Jet Propagation," Ing. Archiv., Vol. V, No. 1, 1934.

6. Weinstein, A. S., Osterle, I. F., Forstall, W.,"Momentum Diffusion from a Slot Jet into a Moving Secondary," Journ. of Applied Mechanics, Vol. XXIII, No. 3, 1956.



Translated for the National Aeronautics and Space Administration under contract No. NASw-2038 by Translation Consultants, Ltd., 944 South Wakefield Street, Arlington, Virginia 22204.